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# CONTRIBUTION TO THE THEORY OF THE HEATED DUCT RADIATOR

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# CONTRIBUTION TO THE THEORY OF THE HEATED DUCT RADIATOR*

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## I. INTRODUCTION

A method is developed whereby without any undue neglect of certain factors the decrease in the mass flow of air and in the drag of the heated radiator, i.e., one under actual operating conditions, as compared with the corresponding values in the cold state, may be simply computed. Although a symmetrical duct radiator has been used to bring out the flow relations, the results apply equally as well to the unsymmetrical radiator shapes usual in airplane construction.

It was Meredith (reference 1) who first pointed out the possible gain in power required to tow the radiator system under the operating condition as compared with the cold condition and showed how the momentum or power gain resulting from the heating of the air by the coolant may be approximately computed with the aid of simple momentum and energy considerations. Since the computation, however, involves rather extensive simplifications, the results may be used tentatively as a first approximation only.

A very clear presentation of the effect of the air forces on a duct radiator, particularly on the radiator cowling, has been given by Weise (reference 2) who explained the change of state of the air in the radiator with the aid of a velocity-pressure diagram. In the present paper simple formulas are derived with the aid of which the lowering in the drag of a radiator due to heating may be computed with an accuracy sufficient for practical requirements. There is first considered the case of the cold radiator and next the effect of the heating of the radiator on the rate of air flow and on the drag.

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*"Beitrag zur Theorie des beheizten Düsenkühlers." From Luftfahrtforschung, vol. 15, no. 10/11, Oct. 10, 1938, pp. 500 to 504.

## II. NOTATION

$v_0$, $\rho_0$, $F_0$, $p_0$, velocity, air density, cross-sectional area, and pressure at duct inlet.

$v_1$, $\rho_1$, $F$, $p_1$, corresponding values immediately in front of radiator block.

$v_2$, $\rho_2$, $F$, $p_2$, corresponding values behind radiator block.

$v_3$, $\rho_3$, $F_a$, $p_0$, corresponding values at duct exit.

See fig. 1.

$\beta = \frac{F}{F_a}$, $\rho_1 \approx \rho_0$, $\rho_3 \approx \rho_2$

$W_{ik}$, internal drag of cold radiator.

$W_i$, internal drag of heated radiator.

$c_w$, drag coefficient of cold radiator.

$\varphi_k$, effective rate of air flow $v_1/v_0$ (equation (6)) through cold radiator.

$\varphi_0$, theoretical flow ratio through cold radiator (equation (6a)).

$\varphi_1$ flow ratio through the heated radiator (equations (14) and (17)).

$\eta_{Di}$, efficiency of diverging passage ahead of radiator block (equations (7) and (7a)).

$\varphi_2 = \frac{v_2}{v_0}$

$\Delta\varphi = 1 - \frac{\varphi_1}{\varphi_k}$

$N$, engine power (hp.).

$Q = 75\,\alpha\,N$, heat quantity to be conducted away, mkg/s.

$$p^* = \frac{p_o}{\frac{1}{2} \rho_o v_o^2}$$

$$\delta = \frac{75 \alpha N}{\frac{1}{2} \rho_o v_o^3 F}$$

$\eta_{th}$, thermodynamic efficiency of the radiator elements.

$\theta$, temperature drop between cooling fluid and air radiator entry.

$\kappa = 1.4$; $c_p = 0.24$; $A = \frac{1}{427}$ (kcal/kgm).

## III. THE UNHEATED DUCT RADIATOR

We first consider the flow through and around an unheated duct radiator. On figure 1 the cross sections and the corresponding velocities, densities, and pressures are indicated. For the cold radiator $v_1 = v_2$. The cross-sectional area $F_o$, which is proportional to the quantity of air flowing through, depends as is known on the cross-section ratio $F/F_a$, on the drag of the enclosed radiator block, and on the remaining flow losses within the radiator. Of these losses that, due to the widening of the flow cross section from $F_o$ to $F$, is of particular importance. The flow may generally be considered only partly as a flow through a diffuser in the usual sense, i.e., as a flow through a diverging passage, since the flow cross section begins to enlarge ahead of the duct entry itself. Up to the stagnation point at the nose of the radiator, the flow may be considered as being free from losses but in the diverging duct in front of the radiator elements losses arise and may be taken approximately into account by introducing a diffuser efficiency as will be shown below. However, losses are also involved in the external flow about the radiator. This flow may be compared with that about the upper side of a more or less steeply inclined wing section. The less the air quantity flowing through for given dimensions of the duct radiator and given speed, i.e., the more divergent the stream lines ahead of the radiator, the greater are the losses of the external flow, particularly those about the leading edge

of the radiator. The flow pattern then corresponds approximately to that about an airfoil set at a large angle of attack; i.e., the flow may, under certain conditions, separate from the outside duct wall and give rise to an external radiator drag that may amount to a considerable portion of the total drag.

We shall next consider the flow at the radiator exit section. Since the flow at the outer duct wall does not entirely separate and the air exit from the radiator takes place without appreciable contraction, the static pressure at the plane of the exit section may be assumed as equal to the external pressure $p_o$. Not considering for the moment the external flow losses, there is obtained the velocity distribution shown by the continuous line in figure 1. From the momentum theorem, neglecting changes in density, the "internal drag" of the cold radiator may be computed as

$$W_{i_k} = \rho_o F v_1 (v_o - v_3)$$

or

$$W_{i_k} = \rho_o F v_o^2 \varphi_k (1 - \beta \varphi_k) \tag{1}$$

where $\varphi_k = \frac{v_1}{v_o}$ and $\beta = \frac{F}{F_a}$.

The internal drag thus depends on the opening ratio of the exit passage and on the flow ratio $\varphi_k$. The latter may be readily computed if the drag coefficient $(c_w)$ of the cold radiator elements is known, for example from tests in the closed duct. The pressure rise ahead of the radiator elements not considering diffuser loss is obtained from the Bernoulli equation

$$p_1 - p_o = \frac{\rho_o}{2} (v_o^2 - v_1^2) \tag{2}$$

For simplicity the diffuser loss is expressed in terms of the total theoretical pressure rise although, as previously mentioned, the diverging flow only partly takes place through the duct itself, and for the actual pressure rise there is obtained

$$p_1 - p = \eta_{D_1} \frac{\rho_o}{2} (v_o^2 - v_1^2) \tag{3}$$

$p_o$

The flow in the duct behind the radiator may be considered as practically without losses, so that

$$p_2 = p_0 + \frac{\rho_0}{2} v_1^2 (\beta^2 - 1) \tag{4}$$

For the pressure drop through the radiator elements, we have

$$p_1 - p_2 = \frac{\rho_0}{2} v_1^2 c_w \tag{5}$$

and after simple substitutions, there is obtained from equations (3), (4), and (5) the rate of flow ratio for the cold radiator

$$\varphi_k = \frac{v_1}{v_0} = \sqrt{\frac{\eta_{D_i}}{c_w + \beta^2 - (1 - \eta_{D_i})}} \tag{6}$$

This ratio thus depends on the diffuser efficiency as well as on $c_w$ and $\beta$. For otherwise equal conditions this efficiency will be somewhat lower for a duct radiator with small inlet section than for one with large inlet section. A contraction of the inlet opening thus leads to a lowering in the air discharge rate (equation (6)) but generally to an increase in the internal drag (equation (1)).

The actual velocity distribution at the radiator exit corresponds approximately to the dotted curve in figure 1. The additional momentum loss is due to the external radiator loss, i.e., to the frictional resistance of the external cowling. This loss will be relatively small, provided the flow separation is not too great, but will increase with decreasing discharge ratio (i.e., for large $c_w$ and $\beta$) on account of the strong flow about the leading edge and, since the internal drag is relatively small, it will form a considerable portion of the total drag. For the same reason, this loss will be somewhat smaller for a radiator with diffuser than for one without a diffuser. The inlet diffuser thus leads to a lowering of the external drag at the expense of the internal drag and the rate of discharge through the radiator. These changes will, however, not be very large, so that the total drag may be considered as sufficiently independent of the inlet cross section while it depends very much on the value $c_w$ of the enclosed radiator block and on the ratio, radiator cross section/exit cross section.

If the rate of flow ratio of the cold radiator is known from measurements, the diffuser efficiency may conversely be determined from it. Denoting by

$$\varphi_o = \sqrt{\frac{1}{c_w + \beta^2}} \tag{6a}$$

the theoretical discharge ratio of the duct radiator is obtained from equations (6) and (6a):

$$\eta_{D_i} = \left(\frac{\varphi_k}{\varphi_o}\right)^2 \frac{1 - \varphi_o^2}{1 - \varphi_k^2} \tag{7}$$

or, since $\varphi_o$ and $\varphi_k$ are, in general, not very different

$$\eta_{D_i} = \left(\frac{\varphi_k}{\varphi_o}\right) \tag{7a}$$

a very simple relation for the expansion efficiency. This efficiency may be determined, for example, from the effective rate of discharge ratio $\varphi_k$ obtained from model tests and from the theoretical discharge ratio $\varphi_o$.

## IV. THE HEATED DUCT RADIATOR

The heating of the radiator elements leads to a decrease in the air density from $\rho_1$ to $\rho_2$ and to an increase in the velocity from $v_1$ to $v_2$. (See fig. 1.)

For the two-control surfaces aa and bb directly in front of and behind the radiator block, the momentum equation gives

$$p_1 + \rho_o v_1^2 = p_2 + \rho_2 v_2^2 + \frac{W}{F} \tag{8}$$

where $W$ denotes the total frontal and frictional resistance of the radiator block[1]. For the cold radiator $(v_1 =$

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[1] $W$ is, of course, not the equivalent to the internal drag of the system: cooler block + duct, according to equation (1).

$v_2$), the pressure difference is a measure of this resistance:

$$W = F(p_1 - p_2) = \frac{\rho_o}{2} v_1^2 \quad c_w \; F$$

For the heated radiator, this is no longer the case since, as follows from equation (8), the change in momentum here also plays an important part. For equal weight of cooling air, the pressure drop in the heated radiator is greater than in the cold one since the momentum at the exit exceeds that at the inlet and moreover the mean dynamic pressure increases to

$$\left(\frac{\rho}{2} v\right)^2_m \cong \frac{1}{2}\left(\frac{\rho_o}{2} v_1^2 + \frac{\rho_2}{2} v_2^2\right)$$

In the drag coefficient $c_w$, there is included the frontal resistance, the frictional loss, for example, in the cooling elements, and the mixing loss at the radiator exit. The frictional drag depends on the Reynolds Number. The small difference in $c_w$ due to the difference in the discharge rate and in the mean air temperature obtained with the heated radiator as compared with the cold radiator, will not, however, be taken into account in the following computation, since, as further computation shows, the change in the discharge rate is very slight and the frictional resistance constitutes only a (not accurately known generally) portion of the total drag.

On substituting the mean dynamic pressure, equation (8) goes over into[2]

$$\Delta p_h = p_1 - p_2 = (\rho_2 v_2^2 - \rho_o v_1^2) + \frac{1}{2}\left(\frac{\rho_o}{2} v_1^2 + \frac{\rho_2}{2} v_2^2\right) c_w \qquad (8a)$$

[2] If the drag coefficient of the hot radiator is defined as

$$c_{w_h} = \frac{\Delta p_h}{\frac{\rho_o}{2} v_1^2}$$

$\Delta p_h$ includes the pressure rise due to the momentum increase and the increase in the mean dynamic pressure. The drag coefficient will then of course differ essentially
(Continued on next page)

On account of the increased pressure drop in the radiator block, the streamlines ahead of the radiator must diverge more strongly than in the case of the cold radiator; i.e., there is a decrease in the rate of air discharge until an equilibrium condition corresponding to equation (8a) is reached. We shall therefore first determine the cooling air discharge rate through the heated radiator and from it the internal drag of the heated radiator.

Equation (8a) contains five unknowns $(p_1, p_2, v_1, v_2, \rho_2)$. If we neglect the very small change in the diffuser efficiency on account of the somewhat decreased rate of discharge, equation (3) remains unchanged. Equation (4) passes over into

$$p_2 = p_0 + \frac{\rho_2}{2} v_2^2 \ (\beta^2 - 1) \tag{9}$$

to which must be added the continuity equation

$$\rho_0 \ v_1 = \rho_2 \ v_2 \tag{10}$$

and finally the energy equation in the following form.

The difference in the total energies at two cross sections immediately in front of and behind the radiator block is equal to the quantity of heat transmitted by the radiator to the air. The frontal and frictional resistance of the radiator gives rise to no loss in the total energy since the work required is directly converted into the friction loss:

$$\rho_0 \ v_1 \ F \ g \left[ \left( i_2 + A \frac{v_2^2}{2 \ g} - i_1 + A \frac{v_1^2}{2 \ g} \right) \right] = Q \tag{11}$$

For the heat content, we have in general

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[2]Continued from page 7
from that of the cold radiator. A similar consideration of the momentum was given by H. Lorenz for the ducted radiator and appreciable deviations were obtained in the results of the measurements, ascribed by Lorenz to the simplified assumptions made. This contradiction is rather to be explained by the fact that through an oversight the increase in momentum was set equal to $\frac{1}{2}(\rho_2 v_2^2 - \rho_0 v_1^2)$ and not to $\rho_2 v_2^2 - \rho_0 v_1^2$ which is the proper expression. (reference 3).

$$i = \frac{\kappa}{\kappa - 1} A \frac{p}{g\,\rho} \qquad \left(A = \frac{1}{427} \text{ kcal/mkg}\right)$$

The quantity of heat Q(kcal/s) may be taken as a fraction of the engine power:

$$Q = A\,\alpha\,75\,N \quad (\text{for water cooling } \alpha \sim 0.50)$$

Equation (11) then becomes

$$\rho_0 v_1 F \left[\frac{\kappa}{\kappa - 1}\left(\frac{p_2}{\rho_2} - \frac{p_1}{\rho_0}\right) + \frac{1}{2}(v_2^{\,2} - v_1^{\,2})\right] = 75\,\alpha\,N \tag{11a}$$

The solution of the system of equations (3), (8a), (9), (10), and (11a) gives a relation between the inlet and outlet velocities where in place of $v_1$ and $v_2$ there are again introduced the nondimensional velocities $\varphi_1 = v_1/v_0$ and $\varphi_2 = v_2/v_0$:

$$\varphi_2 = \frac{\eta_{D_i} + \varphi_1^{\,2}\left(2 - \eta_{D_i} - \frac{c_w}{2}\right)}{\varphi_1\left(\beta^2 + 1 + \frac{c_w}{2}\right)} \tag{12}$$

Finally, there is obtained an equation for the velocity ratio $\varphi_1$

$$\varphi_1 \left[\frac{\kappa}{\kappa - 1} p^* \frac{\eta_{D_i} + \varphi_1^{\,2}\left(2 - \eta_{D_i} - \frac{c_w}{2}\right)}{\varphi_1^{\,2}\left(\beta^2 + 1 + \frac{c_w}{2}\right)} + \right.$$

$$+ \frac{\kappa\,\beta^2 - 1}{\kappa - 1} \; \frac{\eta_{D_i} + \varphi_1^{\,2}\left(2 - \eta_{D_i} - \frac{c_w}{2}\right)}{\varphi_1\left(\beta^2 + 1 + \frac{c_w}{2}\right)} +$$

$$\left. + \varphi_1^{\,2}\left(\frac{\kappa}{\kappa - 1}\eta_{D_i} - 1\right) - \frac{\kappa}{\kappa - 1}(p^* + \eta_{D_i})\right] = \delta \tag{13}$$

where

$$p^* = \frac{p_0}{\frac{1}{2}\rho_0 v_0^{\,2}} \quad \text{and} \quad \delta = \frac{75\,\alpha\,N}{\frac{1}{2}\rho_0 v_0^{\,3} F}$$

Equation (13) can be represented in the form of a family of curves with the parameters $\eta_{D_i}$, $c_w$, $\beta^2$, and $\delta$. It is readily shown that the two middle terms of the expression in brackets, which arise essentially from the squared velocities in equation (11a), are small under normal conditions compared with the other two terms. Furthermore, in the fourth term of the expression in brackets $\eta_{D_i}$ may be neglected in comparison with $p^*$. The resulting expression, neglecting a small term under the root, then becomes

$$\varphi_1 = \varphi_k \left[1 - \frac{1}{2} \frac{\kappa - 1}{\kappa} \frac{\beta^2 + 1 + \frac{c_w}{2}}{\eta_{D_i}^2} \frac{75\ \alpha\ N}{p_0\ F\ v_0} \varphi_k\right] \tag{14}$$

where $\eta_{D_i}$ for a given case is to be determined by equation (7) or estimated. The remaining terms on the right-hand side of the equation may be considered as known. The velocity through the radiator decreases linearly with the quantity of heat carried off $\alpha\ N$. The decrease as compared with the cold radiator amounts to

$$\Delta\varphi = 1 - \frac{\varphi_1}{\varphi_k} \tag{15}$$

The internal drag of the cold radiator is given by equation (1). In the same manner there is obtained for the internal drag of the heated radiator

$$W_i = \rho_0\ F\ v_1\ (v_0 - \beta\ v_2) = \rho^0\ F\ v_0^2\ \varphi_1 (1 - \beta\ \varphi_2)$$

where $\varphi_2$ is determined from equation (12). The decrease in drag therefore amounts to

$$\Delta W_i = W_{i_k} \left(1 - \frac{W_i}{W_{i_k}}\right) = \left(1 - \frac{\varphi_1}{\varphi_k} \frac{1 - \beta\varphi_2}{1 - \beta\ \varphi_k}\right) W_{i_k} \tag{16}$$

Only the internal radiator drag can be readily computed. If the change in the rate of flow in heating the radiator is not very large, the external drag may be considered as unchanged. Equation (16) in that case gives, to a good degree of approximation, the total decrease in the drag due to the heating.

In some cases a different presentation of these results is desirable. The heat given off by a radiator element is generally expressed in the following form

$$Q = F\, v_1\, g\, \rho_o\, c_p\, \eta_{th}\, \theta \text{ (kcal/s)}$$

or

$$Q = 75\, \alpha\, N = 427\, F\, v_o\, g\, \rho_o\, c_p\, \eta_{th}\, \theta\, \varphi_1 \text{ (mkg/s)}$$

where $\eta_{th}$ is the thermodynamic efficiency of the radiator, which is known from tests on the radiator block in the duct as a function of the discharge rate, $\theta$ is the temperature drop between the cooling fluid and the air at the radiator inlet. With this expression for $Q$ or $\alpha N$ the simplified equation (13) gives

$$\varphi_1 = \frac{\varphi_k}{\sqrt{1 + C\,\theta}} \tag{17}$$

$$C = 427\, \frac{\kappa - 1}{\kappa}\, \frac{g\, \rho_o\, c_p\, \eta_{th}}{p_o}\, \frac{\beta^2 + 1 + \frac{c_w}{2}}{\beta^2 + c_w - (1 - \eta_{D_i})}$$

or with

$$\kappa = 1.4, \quad c_p = 0.24$$

$$C = 288\, \frac{\rho_o}{p_o}\, \eta_{th}\, \frac{\beta^2 + 1 + \frac{c_w}{2}}{\beta^2 + c_w - (1 - \eta_{D_i})} \tag{17a}$$

where, except for $\beta$ and $\eta_{D_i}$, $C$ contains only magnitudes that are known from experiment for a ducted radiator. If, in addition, there is known the velocity $\varphi_k$ for the cold ducted radiator $\eta_{D_i}$ may be determined from equation (7) and the velocity for the heated radiator condition ($\theta$) from equations (17) and (17a). The exit velocity $\varphi_2$, the decrease in the discharge velocity and in the drag are given by equation (12), (15), and (16), respectively.

By the method described above, the decrease in the discharge rate and in the internal drag was computed for radiators of various drag coefficients and thermodynamic efficiencies for $\theta = 65^\circ$ (mean value for water cooling) and for $\theta = 115^\circ$, $\eta_{D_i}$ being estimated at 0.85 and $\frac{\rho_o}{p_o} = 0.0000135$ corresponding to normal conditions at 3,000 meters altitude. The results are presented in fig-

ures 2 to 5 as a function of the cross-section ratio $F_a/F$ for $\eta_{th} = 0.6$ and 0.7. As may be seen for $\theta = 115°$ $\eta_{th} = 0.7$ and $F_a/F \approx 0.15$, the condition is reached at which the radiator no longer possesses any internal drag. Since, however, for such duct openings the internal drag may amount to only a very small portion of the total drag, the decrease based on the total drag is relatively small unless special means are provided for keeping the external drag small. The decrease in the discharge rate is practically independent of $c_w$ and in the figures is represented therefore by only one curve. The effect of the decrease in the radiator drag on the power required for towing the duct system may readily be determined by the usual method and will therefore be omitted.

Translation by S. Reiss,
National Advisory Committee
for Aeronautics.

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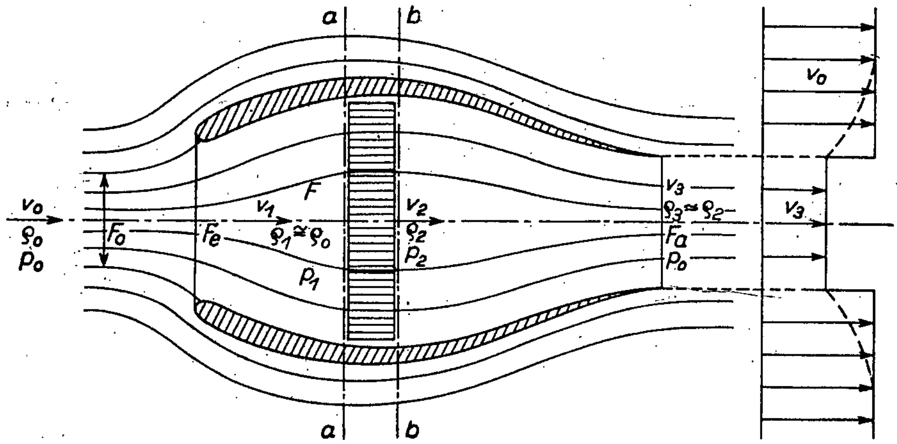


Figure 1.- Flow through and around a ducted radiator.

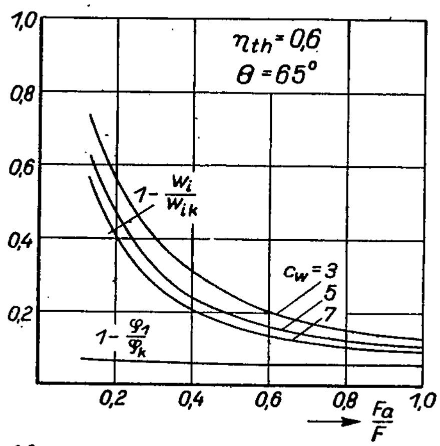


Fig. 2

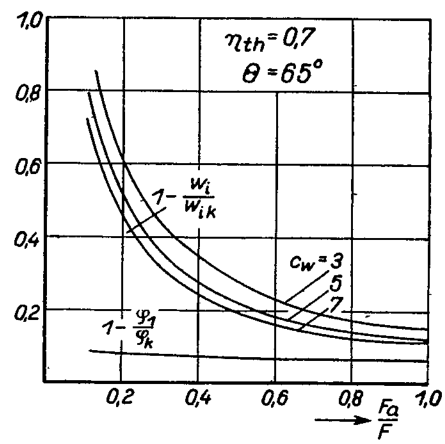


Fig. 4

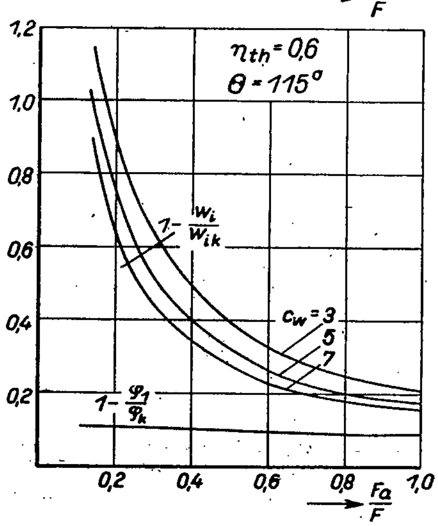


Fig. 3

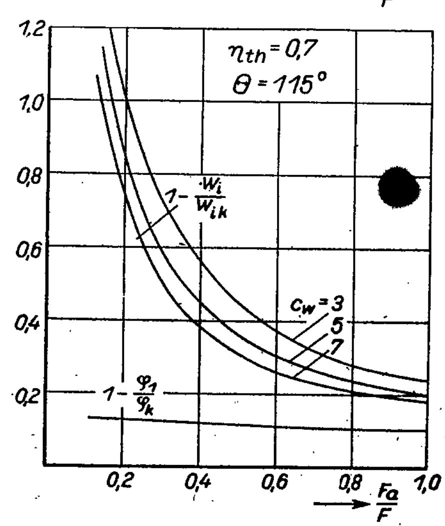


Fig. 5

Figures 2 to 5.- Decrease in the discharge velocity and in the internal drag of a heated radiator in comparison with the unheated condition as a function of the duct opening ratio $F_a/F$ for various drag coefficients of the radiator block($c_w$). $\frac{p_0}{p_0}$ = 0.0000135 (H =3,000 m).